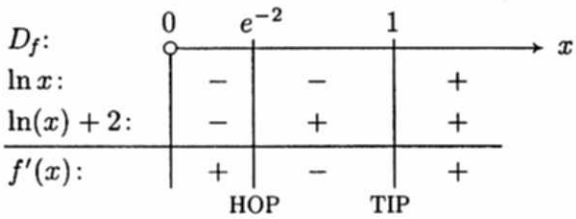
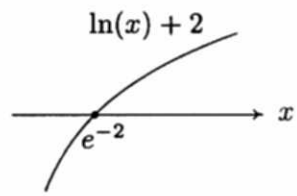
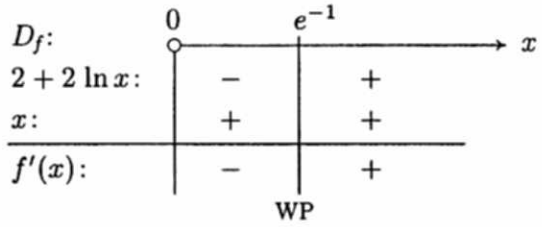
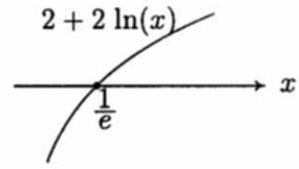
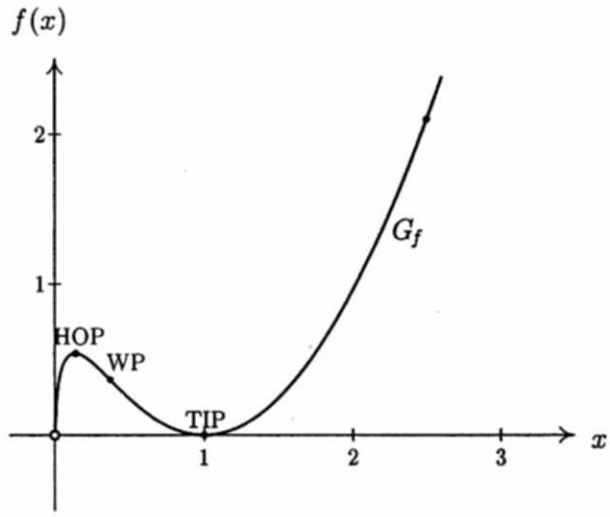
| Nr | | BE |
|---|---|---|
| 8.1 | $f(x) = x \cdot \ln^2 x = x \cdot (\ln x)^2$ , $D_f = \mathbb{R}^+$<br>NSt.: $x_1 = 0 \notin D_f$ , $x_2 = 1$ d.h. eine Nullstelle $x = 1$<br>$\lim\limits_{\substack{x\to 0\\x>0}} x \cdot (\ln x)^2 = „+0 \cdot +\infty" = \lim\limits_{\substack{x\to 0\\x>0}} \frac{(\ln x)^2}{\frac{1}{x}} =$ $(l'H.)$ $\lim\limits_{\substack{x\to 0\\x>0}} \frac{2(\ln x)\cdot\frac{1}{x}}{-\frac{1}{x^2}} = \lim\limits_{\substack{x\to 0\\x>0}} \frac{2\ln x}{-\frac{1}{x}} =$<br>$= (l'H.)$ $\lim\limits_{\substack{x\to 0\\x>0}} \frac{2}{x\cdot\frac{1}{x^2}} = \lim\limits_{\substack{x\to 0\\x>0}} 2x = +0$ , $\lim\limits_{x\to\infty} x^2 \cdot \ln x = „+\infty \cdot +\infty" = +\infty$ | |
| 8.2 | $f'(x) = (\ln x)^2 + x \cdot 2(\ln x) \cdot \frac{1}{x} = (\ln x)^2 + 2\ln x = \ln x \cdot \big(\ln(x) + 2\big)$<br>$f''(x) = \frac{1}{x} \cdot (\ln(x) + 2) + (\ln x) \cdot \frac{1}{x} = \frac{\ln(x) + 2 + \ln(x)}{x} = \frac{2 + 2\ln(x)}{x}$ | |
| 8.3 | **Monotonie:** $f'(x) = 0$ : $\ln x \cdot \big(\ln(x) + 2\big) = 0$ : $\ln x = 0 \iff x = 1$ , $f(1) = 0$<br>$\ln(x) + 2 = 0 \iff \ln(x) = -2 \iff x = e^{-2} = \frac{1}{e^2}$ , $f(\frac{1}{e^2}) = \frac{1}{e^2} \cdot (-2)^2 = \frac{4}{e^2}$<br>  <br><br>  <br><br>$f$ str. mon. zunehmend in $]0;\frac{1}{e^2}]$ sowie in $[1;\infty[$ , $f$ str. mon. abnehmend in $[\frac{1}{e^2};1]$<br>$\Longrightarrow H(\frac{1}{e^2}\,|\,\frac{4}{e^2})$ HOP , $T(1|0)$ TIP | |
| 8.4 | **Krümmung:** $f''(x) = 0$ : $2 + 2\ln x = 0 \iff \ln x = -1 \iff x = e^{-1} = \frac{1}{e}$ , $f(\frac{1}{e}) = \frac{1}{e}$<br>  <br><br>  <br><br>$G_f$ rechtsgekrümmt in $]0;\frac{1}{e}]$ und linksgekrümmt in $[\frac{1}{e};\infty[$ $\Longrightarrow W(\frac{1}{e}\,|\,\frac{1}{e})$ Wendepunkt | |
| 8.5 |  <br> | |